A new frequency domain criterion for RR prediction is proposed based on stability margins in a Nichols chart. The proposed procedure should be further developed to obtain a well-defined RR criterion for flight control system designers applicable to highly maneuverable fighters and to large flexible transport aircraft.

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Linear Quadratic Optimality of Infinite Gain Margin Controllers

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Introduction

INFINITE gain margin controllers (IGMCs) are a class of robust controllers designed with guaranteed stability in the presence of a gain uncertainty factor. The loop transfer functions recovered by them are generally minimum phase with no right half-plane zeros.[1,2] Although seeking robustness, it is also known that the IGMCs deteriorate nominal performance, and so the associated linear quadratic (LQ) cost functions are not necessarily minimum. We consider the cost functions for single input LQ regulators and investigate optimality of the controllers seeking IGM. The problem is stated as follows: Given $\boldsymbol{k}$, an IGMC and a scalar $\rho \geq 1$, does there exist a $\rho \in [1, \infty)$ for which $\rho\boldsymbol{k}$ is LQ optimal? In output feedback setting, this problem is a simple extension to the inverse problem of state feedback LQ regulators.[3] Both frequency- and time-domain optimality criteria are considered for analysis. Similar results for other classes of controllers and cost functions are addressed in the literature.[4,5]

Background

Consider a linear time-invariant n-state single input dynamic system,

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t) \tag{1}$$

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where $\boldsymbol{x}(t)$ and $u(t)$ are the state and control input signals. $\boldsymbol{A}$ and $\boldsymbol{b}$ are real compatible matrices. Whenever $x(0) = x_0$ is nonzero, it is known that the control law $u(t)$ uses a constant gain row vector $\bar{\boldsymbol{k}}$, linear to sensor outputs $\boldsymbol{z}(t) \in \boldsymbol{R}^r$, and regulates desired outputs $\boldsymbol{y}(t) \in \boldsymbol{R}^p$ to a set point (zero, assumed). Using $\boldsymbol{C} \in \boldsymbol{R}^{r \times n}$ and $\boldsymbol{H} \in \boldsymbol{R}^{p \times n}$, we write $\boldsymbol{z}(t)$, $u(t)$, and $\boldsymbol{y}(t)$ as

$$\boldsymbol{z}(t) = \boldsymbol{C}\boldsymbol{x}(t) \tag{2}$$

$$u(t) = -\bar{\boldsymbol{k}}\boldsymbol{z}(t) \tag{3}$$

$$\boldsymbol{y}(t) = \boldsymbol{H}\boldsymbol{x}(t) \qquad \boldsymbol{x}_0 \neq 0 \tag{4}$$

Within this setting, recall the inverse problem of LQ regulators for $\boldsymbol{z}(t) = \boldsymbol{x}(t)$ proposed by Kalman[3] using a cost function J,

$$J = \int_0^\infty \{\boldsymbol{y}'\boldsymbol{y} + u^2\}\,\mathrm{d}t = \int_0^\infty \{\boldsymbol{x}'\boldsymbol{H}'\boldsymbol{H}\boldsymbol{x} + u^2\}\,\mathrm{d}t \tag{5}$$

$\boldsymbol{V}'$ is the transpose of a matrix or a vector $\boldsymbol{V}$. Note that the regulation (or performance) problem considered by Kalman has two categories. In the first case, $\boldsymbol{H}$ is fixed and so is the positive semidefinite (PSD) matrix $\boldsymbol{Q} = \boldsymbol{H}'\boldsymbol{H}$ in J (see Theorem 5 in Ref. 3 for optimality conditions). In the second case, J assumes several $\boldsymbol{H}$ for which a given state feedback control law may be optimal (Theorem 6 in Ref. 3, inverse problem of linear optimal control theory). When $\boldsymbol{y}(t)$ in J is unspecified, the algebraic optimality criteria (Theorem 4 in Ref. 3) determine all possible $\boldsymbol{H}$ for which J is possibly minimum.

In this paper, the inverse problem of linear optimal control theory is presented for controllers seeking IGM. That is, for a stabilizing output feedback gain $\rho\boldsymbol{k}$ with $\rho \geq 1$ in the control law given by

$$u(t) = -\rho\boldsymbol{k}\boldsymbol{x}(t) \tag{6}$$

the problem is to find whether or not $u(t)$ is optimal for any $\rho \in [1, \infty)$. Whenever necessary, $\boldsymbol{k}_x = \bar{\boldsymbol{k}}\boldsymbol{I}_n$, with identity matrix $\boldsymbol{I}_n$ in $\boldsymbol{k} = \bar{\boldsymbol{k}}\boldsymbol{C}$, is used for specializing the state feedback results.[3] The implications of complete controllability and observability of the pairs $[\boldsymbol{A}, \boldsymbol{b}]$ and $[\boldsymbol{k}, \boldsymbol{A}]$ are discussed in Ref. 3. In addition, the following assumptions hold for the output feedback case.

Assumption 1: All nonzero initial conditions $\boldsymbol{x}_0$ satisfying the expected value $E(\boldsymbol{x}_0'\boldsymbol{x}_0) = \boldsymbol{X}$, where $\boldsymbol{X}$ is a set of all positive definite (PD) symmetric matrices, are observable. Thus, $[\boldsymbol{C}, \boldsymbol{A}]$ is completely observable.

Assumption 2: The pair $[\boldsymbol{H}, \boldsymbol{A}]$ is completely observable. Unless $\boldsymbol{x}_0$ in $\boldsymbol{X}$ is zero, this assumption excludes the possibility of $\boldsymbol{y}(t)$ reaching the set point without $u(t)$. This may happen when $\boldsymbol{A}$ is asymptotically stable.

Following the assumptions, the algebraic optimality criteria for $\boldsymbol{k}$ to be LQ optimal are[6]

$$\boldsymbol{P} = \boldsymbol{P}' \text{ (PD)} \tag{7}$$

$$(\boldsymbol{A} - \boldsymbol{b}\boldsymbol{k})'\boldsymbol{P} + \boldsymbol{P}(\boldsymbol{A} - \boldsymbol{b}\boldsymbol{k}) = -(\boldsymbol{Q} + \boldsymbol{k}'\boldsymbol{k}) \tag{8}$$

$$(\boldsymbol{A} - \boldsymbol{b}\boldsymbol{k})'\boldsymbol{S} + \boldsymbol{S}(\boldsymbol{A} - \boldsymbol{b}\boldsymbol{k}) = \boldsymbol{X} \tag{9}$$

$$\bar{\boldsymbol{k}} = \boldsymbol{b}'\boldsymbol{P}\boldsymbol{S}\boldsymbol{C}'(\boldsymbol{C}\boldsymbol{S}\boldsymbol{C}')^{-1} \tag{10}$$

Note that $\boldsymbol{P}$ in Eq. (8) is not unique unless $\boldsymbol{Q} + \boldsymbol{k}'\boldsymbol{k}$ is PSD.[7] The controller $\bar{\boldsymbol{k}}$ in Eq. (10) is optimal for the cost function

$$J = E\left\{\int_0^\infty (\boldsymbol{x}'\boldsymbol{Q}\boldsymbol{x} + u^2)\,\mathrm{d}t\right\} \tag{11}$$

When $\boldsymbol{C} = \boldsymbol{I}_n$, the state feedback gain is $\boldsymbol{k}_x = \boldsymbol{b}'\boldsymbol{P}\boldsymbol{S}\boldsymbol{S}^{-1}$, and Eqs. (8) and (9) are equivalent. Thus, the algebraic optimality conditions simplify to the results of Kalman,[3]

$$\boldsymbol{P} = \boldsymbol{P}' \text{ (unique and PD)} \tag{12}$$

$$\boldsymbol{k}_x = \boldsymbol{b}'\boldsymbol{P} \tag{13}$$

$$\boldsymbol{A}'\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} = -\boldsymbol{Q} + \boldsymbol{P}\boldsymbol{b}\boldsymbol{b}'\boldsymbol{P} \tag{14}$$

The uniqueness of $\boldsymbol{P}$ in Eq. (12) stems from $[\boldsymbol{H}, \boldsymbol{A}]$ being completely observable,[3] instead of $\boldsymbol{Q} + \boldsymbol{k}'\boldsymbol{k}$ in Eq. (8) being strictly PSD. Solution for a controller $\bar{\boldsymbol{k}}$ in Eq. (10) requires parameter optimization,[8] with unknowns $\boldsymbol{P}$ and an uncertainty matrix $\boldsymbol{X}$. Note, when $\bar{\boldsymbol{k}}\boldsymbol{C} = \boldsymbol{b}'\boldsymbol{P}$, any $\boldsymbol{S}$ from Eq. (9) trivially satisfies Eq. (10). We will show that this specific solution applies to IGMCs. Thus, we refer to conditions I as the algebraic optimality criteria for IGMCs in state and output feedback cases:

$$\begin{aligned} &\boldsymbol{P} = \boldsymbol{P}' \text{ (unique and PD)} \\ (I) \qquad &\rho\bar{\boldsymbol{k}}\boldsymbol{C} = \boldsymbol{b}'\boldsymbol{P} \\ &\boldsymbol{A}'\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} = -\boldsymbol{Q} + \boldsymbol{P}\boldsymbol{b}\boldsymbol{b}'\boldsymbol{P} \end{aligned}$$

Main Result

We begin with the inverse problem of state feedback controllers.[3]

Lemma 1: Consider a completely controllable plant $[A, b]$ and a completely observable pair $[\boldsymbol{k}_x, \boldsymbol{A}]$. Then the stabilizing control law $u(t) = -\boldsymbol{k}_x\boldsymbol{x}(t)$ is a nondegenerate optimal control law if and only if $|1 + \boldsymbol{k}_x(j\omega\boldsymbol{I}_n - \boldsymbol{A})^{-1}\boldsymbol{b}|^2 > 1$ is satisfied. In general $\boldsymbol{k}_x$ will be optimal with respect to several $\boldsymbol{H}$, all of them satisfying observability of $[\boldsymbol{H}, \boldsymbol{A}]$ with no common cancellable factors in the rational transfer functions $\boldsymbol{H}(j\omega\boldsymbol{I}_n - \boldsymbol{A})^{-1}\boldsymbol{b}$.

Lemma 1 is the frequency-domain-based optimality condition for $\boldsymbol{k}_x$ simultaneously satisfying the time-domain algebraic optimality conditions I. Suppose $\boldsymbol{k}_x$ is an IGMC not satisfying Kalman's optimality conditions; we will now show that $\rho\boldsymbol{k}_x$ at some ρ in the interval $[1, \infty)$ is indeed optimal. For this purpose, interpretation of Lemma 1 using Bode plots of $h(s)$ is necessary.

Theorem 1: Consider a completely controllable plant $[\boldsymbol{A}, \boldsymbol{b}]$, a completely observable stabilizing state feedback control law $u(t) = -\boldsymbol{k}_x\boldsymbol{x}(t)$ with loop transfer function $h(s) = \boldsymbol{k}_x(s\boldsymbol{I}_n - \boldsymbol{A})^{-1}\boldsymbol{b}$, and a completely observable pair $[\boldsymbol{H}, \boldsymbol{A}]$. Then the control law $u(t)$ is optimal with algebraic optimality conditions I if and only if the magnitude and phase plots of $h(s)$ satisfy

$$|h(j\omega)| > -2\cos[\angle h(j\omega)] \qquad \forall \omega > 0 \tag{15}$$

Proof: Let $h(j\omega) = h_r + jh_i$, $|h(j\omega)|^2 = h_r^2 + h_i^2$, and $h_r = \cos(\angle h)|h(j\omega)|$, where $\angle h$ is $\angle h(j\omega)$, the phase angle for the loop transfer function at all frequencies $\omega > 0$. Because the algebraic optimality conditions I and $|1 + h(j\omega)|^2 > 1$ are equivalent, it suffices to check inequality (15). From the necessary and sufficient conditions (referred by $\Leftrightarrow$) for LQ optimality in Lemma 1, we infer

$$\begin{aligned} &|1 + h(j\omega)|^2 > 1 \\ \Leftrightarrow\ &[1 + h(j\omega)][1 + h(-j\omega)] > 1 \\ \Leftrightarrow\ &1 + 2h_r + h_r^2 + h_i^2 > 1 \\ \Leftrightarrow\ &h_r^2 + h_i^2 > -2h_r \\ \Leftrightarrow\ &|h(j\omega)| > -2\cos(\angle h) \end{aligned}$$

QED

Because $\boldsymbol{k}_x$ is an IGMC and the direction of the vector $(\rho\boldsymbol{h}_r, \rho\boldsymbol{h}_i)'$ for $\rho h(j\omega)$ is unaltered at all $\rho \geq 1$, the invariance of $-2\cos(\angle\rho h)$ at any $\rho \geq 1$ is inferred. Suppose now the IGMC $\boldsymbol{k}_x$ is not optimal and $-2\cos(\angle h)$ is positive over some ω [otherwise inequality (15) is trivially satisfied]; then inequality (15) can be satisfied by selecting any ρ given by

$$\rho > \rho^* = \sup_{\omega > 0} \frac{-2\cos[\angle h(j\omega)]}{|h(j\omega)|}$$

Thus, Theorem 1 for IGM state feedback gains is specialized as follows.

Corollary 1 (state feedback case): Let $\boldsymbol{k}_x$ be a nonoptimal IGMC with loop transfer function $h(s) = \boldsymbol{k}_x(s\boldsymbol{I}_n - \boldsymbol{A})^{-1}\boldsymbol{b}$ and ρ^* given by

$$\rho^* = \sup_{\omega > 0} \frac{-2\cos[\angle h(j\omega)]}{|h(j\omega)|}$$

Then $\rho\boldsymbol{k}_x$ for all $\rho > \rho^*$ is optimal and guarantees the algebraic optimality conditions I.

Proof:

$$\begin{aligned} \rho > \rho^* = &\sup_{\omega} \frac{-2\cos(\angle h)}{|h(j\omega)|} \\ &\Leftrightarrow \rho|h(j\omega)| > -2\cos(\angle h) \\ &\Leftrightarrow |\rho k_x(sI_n - A)^{-1}b| > -2\cos(\angle h) \\ &\Leftrightarrow \text{by Theorem 1}, \rho k_x \text{ is optimal} \end{aligned}$$

Because Theorem 1 and the algebraic optimality conditions I are equivalent, ρk_x satisfies conditions I. QED

Corollary 2 (output feedback case): Consider a completely controllable and observable system $[\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{C}]$. Let $\bar{\boldsymbol{k}}\boldsymbol{C}$ be a stabilizing nonoptimal IGMC with no cancellable factors in the loop transfer function $h(s)$ given by

$$h(s) = \bar{k}C(sI_n - A)^{-1}b$$

Let ρ^* for $h(s)$ satisfy

$$\rho^* = \sup_{\omega > 0} \frac{-2\cos[\angle h(j\omega)]}{|h(j\omega)|}$$

Then for all $\rho > \rho^*$, $\rho\bar{\boldsymbol{k}}\boldsymbol{C}$ is optimal and satisfies the algebraic optimality conditions I.

Proof: The proof is similar to Corollary 1. However, loop transfer function $h(s)$, algebraic optimality conditions I, and $\rho > \rho^*$ are now defined using the controller $\rho\bar{\boldsymbol{k}}\boldsymbol{C}$. QED

In a design setting, the pole-zero excess of the minimal phase loop transfer functions $h(s)$ by optimal IGMCs are restricted to either 1 or 2 (Ref. 2). For $h(s)$ with pole-zero excess of 2, the intersection of the asymptotes in root locus discussion must be centered at the open left-half plane. In cases where $h(s)$ exceeds 3, note that the asymptotes from root locus technique intersect the imaginary axis in the direction of the right-half plane. Thus, $h(s)$ cannot have IGM.

Remark 1 (design inference): Let the rational transfer functions $\boldsymbol{C}(s\boldsymbol{I}_n - \boldsymbol{A})^{-1}\boldsymbol{b}$ construct a loop transfer function $\boldsymbol{k}(s\boldsymbol{I}_n - \boldsymbol{A})^{-1}b$ or $\boldsymbol{k}_x(s\boldsymbol{I}_n - \boldsymbol{A})^{-1}b$ with pole-zero excess 1 or 2. Let the pole and zero locations p_i and z_j of a minimal phase $h(s)$ satisfy the condition

$$\sum_{i=1}^{n} p_i - \sum_{i=1}^{m} z_i < 0$$

whenever $n - m = 2$. If $\boldsymbol{k}_x$ for a state feedback case, or $\boldsymbol{k}$ for an output feedback case, can assign z_j in the open left-half plane, then the controllers $\rho\boldsymbol{k}$ or $\rho\boldsymbol{k}_x$ can be made optimal at $\rho > \rho^*$ as discussed in Corollaries 1 and 2.

Remark 2 (optimal controller for regional pole constraints): Suppose $\boldsymbol{k}_x$ or $\boldsymbol{k}$ in Remark 1 arbitrarily assigns z_j; then selection and assignment of the zero locations z_j determine approximate open left-half plane regions for the eigenvalues of $(\boldsymbol{A} - \boldsymbol{b}\boldsymbol{k})$.

Existence, Uniqueness, and Computation of Symmetric Matrices *P* and *Q*

Existence and uniqueness of $\boldsymbol{P}$ for IGMC are obvious from Lemma 1, because $\boldsymbol{Q} = \boldsymbol{H}'\boldsymbol{H}$ is PSD,[7] and $[\boldsymbol{H}, \boldsymbol{A}]$ is completely observable.[3] To compute $\boldsymbol{P}$ and $\boldsymbol{Q}$ in both state and output feedback cases, we follow the next result.[9]

Lemma 2: For a controllable system $[\boldsymbol{A}, \boldsymbol{b}]$, there exists a PD matrix $\boldsymbol{P} = \boldsymbol{P}'$ and a PSD matrix $\boldsymbol{Q} = \boldsymbol{Q}'$ if and only if $\boldsymbol{K} = \rho\boldsymbol{k}_x$ (or $\rho\boldsymbol{k}$) satisfies, 1) $(\boldsymbol{A} - \boldsymbol{b}\boldsymbol{K})$ is stable, 2) $\boldsymbol{K}\boldsymbol{b} > 0$, and 3) $-(\boldsymbol{b}'\boldsymbol{A}'\boldsymbol{K}' + \boldsymbol{K}\boldsymbol{A}\boldsymbol{b}) + \boldsymbol{b}'\boldsymbol{K}'\boldsymbol{K}\boldsymbol{b} \geq 0$.

Condition 1 for state and output feedback cases is trivially satisfied because we assume a stabilizing IGMC. Because $\boldsymbol{K} = \boldsymbol{b}'\boldsymbol{P}$ and $\boldsymbol{P}$ is PD, condition 2 implies $\boldsymbol{b}'\boldsymbol{P}\boldsymbol{b} > 0$, which is also true because $\boldsymbol{P}$ is PD. Condition 3 comes from Eq. (14) to guarantee a PSD $\boldsymbol{Q}$. By conditions 1 and 2, P for controller $\boldsymbol{K}$ is

$$\boldsymbol{P} = \boldsymbol{K}'(\boldsymbol{K}\boldsymbol{B})^{-1}\boldsymbol{K} + \boldsymbol{Y} \tag{16}$$

where

$$\boldsymbol{Y}\boldsymbol{b} = 0 \tag{17}$$

If condition 3 holds, $\boldsymbol{Y} = \boldsymbol{Y}'$ computing a PSD $\boldsymbol{Q}$ is

$$\boldsymbol{Q} = -\boldsymbol{A}'\boldsymbol{K}'(\boldsymbol{K}\boldsymbol{b})^{-1}\boldsymbol{K} - \boldsymbol{K}'(\boldsymbol{K}\boldsymbol{b})^{-1}\boldsymbol{K}\boldsymbol{A} + \boldsymbol{K}'\boldsymbol{K} - \boldsymbol{A}'\boldsymbol{Y} - \boldsymbol{Y}\boldsymbol{A} \tag{18}$$

Examples

Consider the short-period dynamics of an aircraft with state variables $x_1(t)$ (angle of attack, degree), $x_2(t)$ (pitch rate, degree/second) and a control input $u(t)$ (the elevator deflection, degree). The controllable pair $[A, b]$ is

$$\boldsymbol{A} = \begin{bmatrix} -0.7520 & 1.001 \\ 0.07896 & -0.8725 \end{bmatrix}, \qquad \boldsymbol{b} = \begin{bmatrix} -0.0631 \\ -3.3990 \end{bmatrix}$$

For $\boldsymbol{b}$, the $\boldsymbol{Y}(\eta)$ in Eq. (17) with a free parameter η is

$$\boldsymbol{Y}(\eta) = \begin{bmatrix} y_1 & y_2 \\ y_2 & 1 \end{bmatrix}\eta, \qquad y_1 = 2901.64054239365$$
$$y_2 = -53.86687797147 \tag{19}$$

State Feedback Case

Based on the control law $u(t) = -\boldsymbol{k}_x\boldsymbol{x}(t)$ with sensors $\boldsymbol{z}(t) = \boldsymbol{x}(t)$, the loop transfer function $h(s)$ can have three possible IGMCs, namely, $\boldsymbol{k}_{x,j}$, $j = 1, 2, 3$ for three different zero (z_1) locations of $h(s)$[10]:

$$\boldsymbol{k}_{x,1} = [1, -1.5531], \qquad z_1 \in (-0.5247, 0)$$

$$\boldsymbol{k}_{x,2} = [0.01, -0.1879], \qquad z_1 \in (-1.0998, -0.5247)$$

$$\boldsymbol{k}_{x,3} = [-1.2720, -1], \qquad z_1 \in (-\infty, -1.0998)$$

All of these controllers satisfy optimality conditions in Theorem 1. The symmetric matrices $\boldsymbol{P}$ by Eq. (16) and $\boldsymbol{Q}$ by Eq. (18) for each controller $\boldsymbol{k}_{x,j} = \boldsymbol{b}'\boldsymbol{P}$ are computed using $\boldsymbol{Y}(10^{-4})$, $\boldsymbol{Y}(10^{-6})$, and $\boldsymbol{Y}(10^{-3})$. $\boldsymbol{Q}$ in each case is PD.

Output Feedback Case

Here we consider the preceding state feedback gains and assume either an angle-of-attack sensor $(z = x_1)$ or a pitch rate sensor $(z = x_2)$ is failed. In both cases, the minimal phase loop transfer functions $h(s)$ with pole-zero excess of 1 guarantee an IGMC. Suppose $\boldsymbol{k}_{x,j} = [k_{a,j}, k_{q,j}]$; it is inferred that the IGMCs $\boldsymbol{k}_j = [0, k_{q,j}]$ in the pitch rate sensor loop are all optimal by inequality (15). Positive definite $\boldsymbol{P}$ and $\boldsymbol{Q}$ matrices for each controller $\boldsymbol{k}_j = \boldsymbol{b}'\boldsymbol{P}$, $j = 1, 2, 3$, are computed using $\boldsymbol{Y}(10^{-3})$, $\boldsymbol{Y}(10^{-5})$, and $\boldsymbol{Y}(10^{-4})$. However, when it fails, $\boldsymbol{k}_4 = [-1.2720, 0]$ is the only stabilizing IGMC by the angle-of-attack sensor. Moreover, gain $\boldsymbol{k}_4$ is not optimal by inequality (15) (see Fig. 1). However, it is made optimal at $\rho > \rho^* = 1320.9$ (see Fig. 2). Suppose we choose $\rho = 1330$; then $\boldsymbol{K} = \rho\boldsymbol{k}_4$ in Lemma 2 proves the existence of $\boldsymbol{P}$ and $\boldsymbol{Q}$. Because the constant gain for the angle-of-attack sensor does not seem to add adequate damping, $\rho = 1330$ suggests a cheap optimal control law. In this case, a PD or a PSD $\boldsymbol{Q}$, though it exists, is not simple to compute.

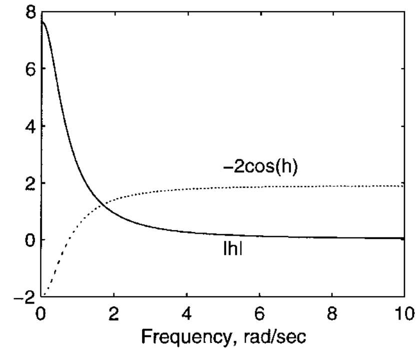

Fig. 1 Nonoptimal IGMC k_4.

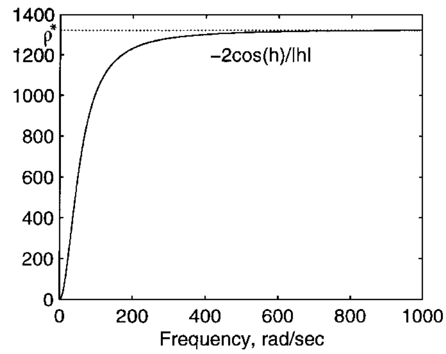

Fig. 2 Optimal IGMC ρk_4, $\rho > \rho^*$.

Conclusions

In this Note, a class of constant gain feedback control laws seeking IGM is considered for investigating the inverse problem of LQ optimal regulators. Using Bode plots of a loop transfer function, it is shown that, in both state and output feedback cases, all single input IGMCs are indeed LQ optimal. Algebraic optimality criteria for these controllers are also presented. An aircraft control problem illustrates state and output feedback settings.

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